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Abstract. The effects of isospin-symmetry breaking on proton emission following β -decay to the isobaric analog state are discussed in detail. Of particular importance is the mixing with a dense background of lower isospin states, whose properties are not well known. The possibility of observing $T=4$ states in even-even, $N=Z$ nuclei, which is viable if the decay proceeds via isospin-forbidden particle emission, is also discussed.

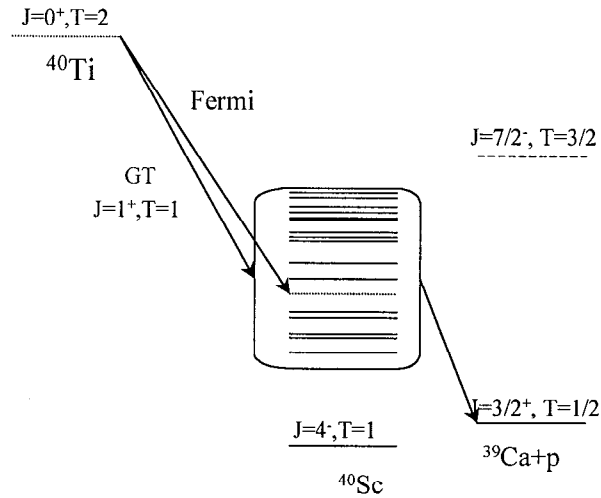
INTRODUCTION

The study of nuclei at the extreme limits of stability has recently become a central focus in nuclear structure research. A powerful tool used in these studies is β -delayed proton emission. Near the proton drip-line, the β -endpoint is quite large, and the β -decay proceeds through a variety of excited states in the daughter nucleus. In turn, these states are generally proton-unbound. Because of the relative ease in detecting and measuring the energy of the emitted protons, it is possible to construct a detailed picture of the structure of the daughter nucleus. In addition, it is also possible to determine the branching ratios for the β -decay, and, hence, measure the $B(GT)$ values. Some particularly important applications are ^{37}Ca [1] and ^{40}Ti [2,3] (the analogs of ^{37}Cl and ^{40}Ar , respectively), which provide a mechanism for calibrating solar-neutrino detectors.

Because of its approximate conservation, isospin is a powerful spectroscopic tool that can often be exploited to map out the structure of nuclei. In nuclei near the proton drip-line, the β -decay to the isobaric analog state is permitted because of the large β -endpoint energy ($\sim 8\text{-}15$ MeV). In many cases, however, the analog state is bound relative to the emission of a proton into the $T'=T-1/2$ state. If isospin were a conserved quantity, the isobaric analog state could not decay by particle emission at all. However, because of the presence of the Coulomb interaction and other weaker isospin-nonconserving (INC) components of the nucleon-nucleon interaction, isospin symmetry is broken and each state can have admixtures of up to $\Delta T = \pm 2$. Consequently, the analog state can decay via the emission of protons to $T'=T-3/2$ states. A schematic of this type of decay is illustrated for ^{40}Ti in Figure 1, where the $J=0$, $T=2$ ground state of ^{40}Ti β -decays to $J=1$, $T=1$ (via Gamow-Teller transitions) states in ^{40}Sc that are all unbound relative to the $T=1/2$ ground state of ^{39}Ca . In addition, the β -decay proceeds to the analog $J=0$, $T=2$ (Fermi transition) state in ^{40}Sc , which is bound relative to the $J=7/2$, $T=3/2$ excited state in ^{39}Ca . Since isospin is not a

conserved quantity, the isobaric analog state in ^{40}Sc has admixtures of $T=1$ states,

FIGURE 1. Schematic of the decay of ^{40}Ti .



while the low-lying $T=1/2$ states in ^{39}Ca have admixtures of $T=3/2$. Hence, proton emission in the analog proceeds via two processes: (1) the decay of the small $T=1$ admixtures in the analog to the $T=1/2$ component of the final state; and (2) the decay of the $T=2$ analog to small admixtures of $T=3/2$ in the final state. This is schematically illustrated in Figure 2, where the dotted lines represent the analog $T=2$ state and the $T=3/2$ state in ^{39}Ca . The solid lines represent the $T=1$ states in ^{40}Sc and the $T=1/2$ ^{39}Ca ground state.

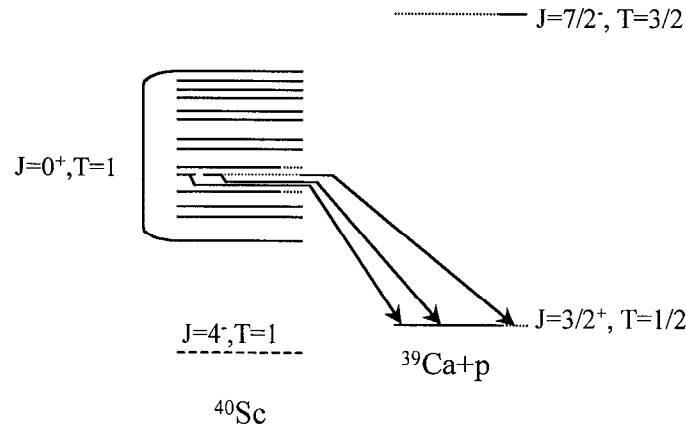


FIGURE 2. Schematic illustration of the isospin-forbidden decay of the analog state.

In the next section, a detailed description of the process behind isospin-forbidden proton emission is given. It will be seen that of particular importance is the role played by the background $T=1$ states, especially their excitation energy relative to the analog state. Afterwards, is a short discussion on the effects of isospin mixing on the β -decay. In addition to applications regarding β -delayed proton emission, the possibility of observing high-lying $T=4$ states in even-even, $N=Z$ nuclei, such as ^{40}Ca , which is possible if proton and neutron emission is isospin forbidden, is discussed. Concluding remarks are gathered in the final section.

DETAILED PICTURE OF ISOSPIN-FORBIDDEN PROTON EMISSION

In this section, I will discuss the physics required for a quantitative description of isospin-forbidden particle emission. Instead of a specific example relating to β -delayed proton emission, I will focus on a systematic study of isospin-forbidden resonances carried out on even-even, $N=Z$ sd -shell nuclei [4]. In these experiments, the compound nucleus shares the same feature exhibited in Figure 2; namely that the state under investigation lies at a relatively high excitation energy and is embedded in a background of $T=1$ states. From first-order perturbation theory, the mixing amplitude between the analog and the $T=1$ states is given by

$$\alpha = \frac{\langle \psi_{T=1} | V_{INC} | \psi_T \rangle}{E_T - E_{T=1}}, \quad (1)$$

where V_{INC} represents the isospin-nonconserving (INC) interaction, and E_T is the excitation energy of the state with isospin T . In general, the matrix elements of V_{INC} are found to be of the order 10-50 keV. From Eq. (1), it is immediately clear that the background $T=1$ states play a crucial role; especially those with excitation energies within 100 keV of the analog state. Unfortunately, even the best theoretical estimates of excitation energies (e.g., from large-basis shell-model calculations) have an uncertainty of the order 200-500 keV. Consequently, in the absence of experimental information about the excitation energies of the $T=1$ states, a quantitative description of the effects of isospin mixing is difficult to assess. The uncertainty imposed on a quantitative picture by the lack information about the background $T=1$ states was investigated in Ref. [5], and here I will recount the main features.

The experimental observable was the nuclear-structure spectroscopic factor Θ extracted from the resonance width Γ via R -matrix theory

$$\Gamma = 2P\gamma_{sp}^2\Theta^2, \quad (2)$$

where $\gamma_{sp}^2 = \hbar^2/R_c^2$, with $R_c \approx 1.4(A_1^{1/3} + A_2^{1/3})$ representing the channel radius. The penetrability P_l for a final state with relative orbital angular momentum l is given by

$$P_l = \frac{kR_c}{F_l^2(kR_c) + G_l^2(kR_c)}, \quad (3)$$

where k is the wave number of the emitted particle and F_l and G_l are regular and irregular free-particle solutions to the Schrödinger equation.

In the theoretical study of Ref. [5], the spectroscopic factor Θ was evaluated via perturbation theory. Hence, Θ represents a sum of the contributions due to the mixing of $T=1/2$ states in the compound nucleus as well as to contributions due to mixing with $T=1$ and 2 states in the target nucleus. The situation is similar to that in Figure 2, and is illustrated in Figure 3 for the specific case of ^{20}Ne . As mentioned above,

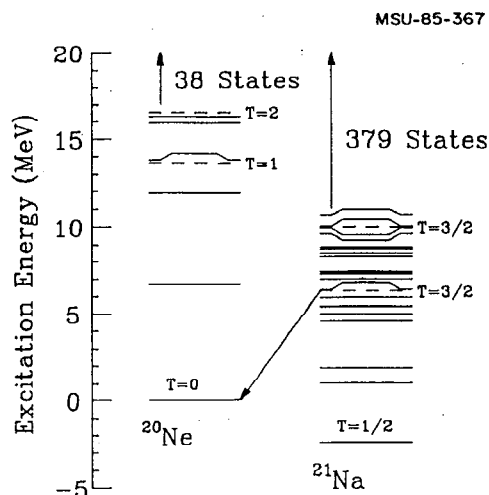


FIGURE 3. Level scheme in ^{20}Ne and ^{21}Na illustrating the relative location of $T=1$ and 2 states in the target nucleus and the $T=1/2$ states in the compound nucleus.

the excitation energy of the background $T=1/2$ states is unknown. The effect of the location of the $T=1/2$ states is illustrated in Figure 4, where Θ is evaluated while shifting the entire $T=1/2$ spectrum by an amount $\delta\epsilon$. In Figure 4, it is seen that over a reasonable range of the shift $\delta\epsilon$, say ± 300 keV, the magnitude of Θ changes by approximately a factor of ten, and reaches a maximum when one of the background $T=1/2$ states is “accidentally” degenerate with the $T=3/2$ state. Given this sensitivity, in Ref. [5] the best estimate of Θ was obtained by taking the average over the range $\delta\epsilon = \pm 500$ keV, with a “theoretical” uncertainty given by the variance over this range. The calculated spectroscopic factors are compared with experimental values on the right-side of Figure 4. In the figure, the solid and open squares represent the experimental data [4] (the experimental errors are approximately the size of the symbols), while the crosses and error bars represent the “best” theoretical estimates. In the figure, it is seen that although in each case, the spectroscopic factor cannot be estimated to better than a factor of two, the range generally encompasses the experimental data. The noted exception being for $A=37$, where the shell-model space used in the calculation is most likely inadequate.

A reasonable estimate of the lifetime for the decay of the analog state by isospin-forbidden proton emission can be obtained using Eq. (2) and estimating the spectroscopic factor to be of the order 10^{-2} . With a separation energy of approximately

5 MeV, we have $2P\gamma_p^2 \approx 1-2$ MeV and $\Gamma \sim 10$ eV (or $T_{1/2} \sim 10^{-16}$ s). On the other hand, typical widths associated with γ -emission are of the order $10^{-7} - 10^{-3}$ eV. Hence, even though it is isospin forbidden, the proton emission is often the dominant decay mode.

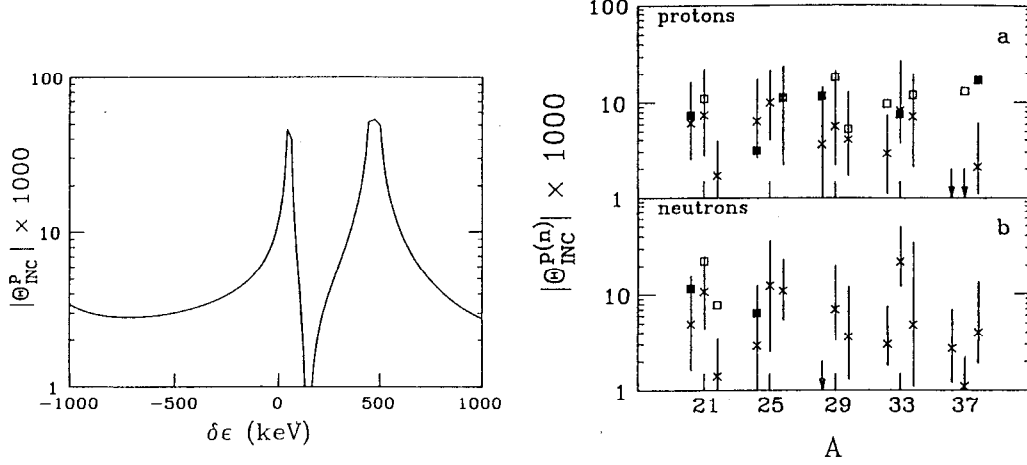


FIGURE 4. On the left, Θ evaluated as a function of the shift $\delta\epsilon$ in the background $T=1/2$ spectrum. On the right, the theoretical estimates of Θ (crosses with error bars) are compared with experimental data (solid and open squares) [4].

EFFECTS OF ISOSPIN MIXING ON BETA-DECAY

Another question that can be raised is whether isospin-symmetry violation will significantly alter the decay rate for the Fermi transition. Because of isospin mixing, the Fermi strength will generally be distributed over a range of background $T=1$ states. Given that the decay rate, Γ_β , for allowed β -decay is proportional to W^5 , where W is the β -endpoint, a distribution of the strength over an energy range could lead to a change in the total Fermi decay rate, and affect estimates for the half-life. Typically, the mixing matrix elements are generally less than 50 keV. Hence, substantial mixing will occur over a narrow range of background states. In Ref. [2] the affect of isospin-mixing was estimated using a two-level mixing model. It was found that the *total* Fermi decay rate differed from the “pure” transition by no more than 0.2% even in the case of maximal mixing with a large mixing-matrix element of 100 keV. It should be noted, however, that in this case the decay rate, and, hence, the branching ratio, to each of the individual states are from the “pure” transition. In general, one expects that the Fermi strength will be distributed over a range of only \pm a few hundred keV around the analog state.

I conclude the discussion of isospin mixing on β -decay studies involving proton emission with some remarks regarding the Fermi transition in odd nuclei. In the example in shown in Figure 1, the Fermi and Gamow-Teller transitions are separated by angular momentum selection rules, i.e., the transitions to $J=1$, $T=1$ states are pure Gamow-Teller. The same is not true for odd- A nuclei, where because of isospin

mixing, the Fermi strength will be distributed to all states with the same angular momentum as the analog state. As a consequence, some β -transitions to T=1 states will contain both Gamow-Teller and Fermi components. Since the Fermi matrix element is generally larger than the Gamow-Teller matrix element even small admixtures of the Fermi transitions can enhance the decay rate to these states. This can be very important for experiments comparing measured and theoretical B(GT) values to assess the magnitude of the quenching of the Gamow-Teller strength.

T=4 STATES IN EVEN-EVEN, N=Z NUCLEI

A goal in physics is to probe the response of nuclear systems at the extreme limits of existence; in particular, at high spin, high excitation energy, or a large proton or neutron excess (i.e., high isospin). Of interest is also the location and formation of the analog of these high isospin states in stable nuclei, for example in even-even, N=Z nuclei. The observation of these states poses several challenges in that they are expected to lie at a high excitation energy, e.g., 20-35 MeV. The viability of observing these high-isospin states lies in the lifetime of the state, and, in particular, whether they are bound relative to isospin-allowed particle emission. If they are, the fact that the decay proceeds through isospin-forbidden channels offers the promise that the states will be long-lived, and, hence, have a width narrow enough to be resolved experimentally. This is especially important given that one is searching for a single state buried in a dense background, whose total density is expected to exceed several thousand per MeV.

Here, I address the question as to whether it is feasible to observe T=4 states in even-even, N=Z nuclei. These T=4 states can be formed with the $(\alpha, {}^8\text{He})$ transfer reaction on $T_z=-2$ nuclei [here $T_z=(Z-N)/2$], and can be performed with the S800 spectrometer at the National Superconducting Cyclotron Laboratory at Michigan State. Viable targets include, ${}^{36}\text{Ar}$, ${}^{44}\text{Ca}$, ${}^{46}\text{Ti}$, ${}^{52}\text{Cr}$, ${}^{56}\text{Fe}$, ${}^{60}\text{Ni}$, and ${}^{64}\text{Zn}$. In this work, I will focus on ${}^{40}\text{Ca}$ (${}^{44}\text{Ca}$ target) to illustrate the principal features.

Predictions for the Binding Energy of the T=4 State

The primary questions to be answered are: (1) what is the excitation energy of the T=4 state? and (2) is it bound relative to isospin-allowed particle emission? As mentioned above, shell-model predictions for excitation energies generally have uncertainties of several hundred keV. The primary source of this uncertainty lies in the treatment of the isospin-conserving part of the strong interaction. On the other hand, the isospin-nonconserving (INC) components (the Coulomb and isotensor part of the strong interaction) are considerably weaker, and can be reasonably addressed with perturbation theory. In order to minimize the uncertainties due to the isospin-conserving part of the strong interaction, I make use of the isobaric-mass-multiplet equation (IIME), namely that for the elements of an isospin-multiplet, the binding energy (BE) in each member is given by [6]

$$BE(T_i) = a + bT_i + cT_i^2, \quad (4)$$

where the a -coefficient is due to the isospin-conserving part of the strong interaction, and the b - and c -coefficients are due to the INC interaction.

The utility of Eq. (4) is that if the binding energy of any one member of the isospin multiplet is known experimentally, the binding energy of all the other members can be predicted using theoretical estimates for the b - and c -coefficients. In Ref. [7], shell-model V_{INC} Hamiltonians were developed for a variety of model spaces ranging from the p -shell to the fp -shell. These V_{INC} interactions were found to reproduce experimental b - and c -coefficients at the level of 25 keV and 15 keV, respectively.

In order to estimate the binding energy of the $T=4$ state in ^{40}Ca , a shell-model calculation was carried out for the b - and c -coefficients using the INC interaction of Ref. [7] within the $0d_{3/2}$ - $0f_{7/2}$ model space. These were then added to the experimental binding energy [8] of the $T_z=-4$ analog state, i.e., ^{40}S . The same procedure was then followed to determine the binding energies of the $T=5/2$ and $7/2$ states in both ^{39}Ca and ^{39}K . The binding energies and relative excitation energies are shown in Table 1. The quoted uncertainties arise from experimental errors (~ 230 keV for ^{40}S) and the theoretical uncertainties of 25 keV and 15 keV for the b - and c -coefficients, respectively. In order to provide a further overall check, the same procedure was followed to predict the excitation energies of the experimentally known $T=1$ and 2 states in ^{40}Ca and the $T=3/2$ state in ^{39}K . Agreement was achieved at the level of 10-20 keV for each of these levels.

TABLE 1. Predicted binding and excitation energies for states in ^{40}Ca , ^{39}Ca , and ^{39}K and the experimental binding energy of the $T=3$ state in ^{38}Ar .

Nucleus	T	Binding Energy (MeV)	Excitation Energy (MeV)
^{40}Ca	4	306.936(412)	35.116(412)
^{39}Ca	5/2	318.086(150)	15.638(150)
^{39}Ca	7/2	306.134(271)	27.589(271)
^{39}K	5/2	310.926(150)	15.485(150)
^{39}K	7/2	298.982(271)	27.429(271)

With the predicted binding energies, we are now in the position to determine if the $T=4$ state will be sufficiently narrow so as to be observable. From Table 1, it is apparent that the $T=4$ state is bound relative to isospin-allowed proton and neutron emission. By comparing the predicted binding energy with the experimental binding and excitation energies tabulated in Refs. [8,9], the $J=0$, $T=4$ state in ^{40}Ca is found to be unbound to the emission of two protons by 1.6(4) MeV to the $T=3$ state in ^{38}Ar . Note that this decay mode is isospin allowed.

Estimates of the Width

The width of the $T=4$ state will have two components. The first is associated with the lifetime for particles to escape, which, following the notation used in descriptions of the giant-dipole resonance, is denoted by Γ^\uparrow . The second component, Γ^\downarrow , is due to the fact that the $T=4$ state will mix with nearby $T=2$ and 3 states, thereby spreading over an energy window comparable with the mixing matrix elements.

Spreading width Γ^\downarrow

The spreading width can be estimated following the arguments in Ref. [10] giving

$$\Gamma^\downarrow \approx 2\pi v^2 \rho_{T=2,T=3}(E_{T=4}), \quad (5)$$

where v^2 is the square of the average matrix element of V_{INC} , $\rho_T(E_{T=4})$ is the density of T=2 and 3 states at an excitation energy $E_{T=4}$ that mix with the T=4 state. In general, the mixing matrix element is of the order 10 keV. In practical application of Eq. (5), ρ_T is the most difficult quantity to estimate reliably. One possible recourse is to perform a shell-model calculation, and simply count the number of levels in the vicinity of the T=4 state. Unfortunately, a calculation utilizing the full 4p-4h excitations in the *sd-pf* shell is not feasible. Consequently, a corrected Fermi-gas estimate will be employed with the proviso that it most likely represents a significant over counting the relevant levels. The Fermi-gas formula for the density of levels as a function of excitation energy E is [10]

$$\rho(E) = (2J+1) \frac{1}{48} \sqrt{\frac{6}{\pi}} a \left(\frac{a\hbar^2}{I_{rig}} \right)^{3/2} \frac{\exp(2\sqrt{aE'})}{[aE']^{1/4}}, \quad (6)$$

where $a \sim A/10$ is the level density parameter, I_{rig} is the rigid body moment of inertia, and $E' = E - J(J+1)\hbar^2/2I_{rig}$. As mentioned above, at the excitation energy of the T=4 state, approximately 35 MeV, Eq. (6) vastly overestimates the density of 4p-4h states that can mix. To account for this overcounting, the excitation energy E' in Eq. (6) is further shifted by 18.5 MeV, which corresponds to the difference in the excitation energies between the T=4 state and 4p-4h, T=2 states obtained from a shell-model calculation. The shell-model calculation was carried out using the interaction of Ref. [11] while limiting the 4p-4h excitations to the $0d_{3/2}, 1s_{1/2}, 0f_{7/2}, 1p_{3/2}$ model space. This leads to $\rho \sim 880 \text{ MeV}^{-1}$, and, hence, $\Gamma^\downarrow \sim 500 \text{ keV}$.

Escape width Γ^\uparrow

A simple estimate of the escape width can be obtained from Eq. (2). The maximum energy for an emitted proton is of the order 9.5 MeV, which for an $l=1$ proton gives $2P\gamma_p^2 \approx 6 \text{ MeV}$. Note that since the T=4 and 5/2 states are of different parity, the l of the emitted proton must be odd. As was seen in the previous subsection, the T=4 state spreads over an energy range of approximately 100-200 keV, therefore, a reasonable estimate of the spectroscopic factor associated with the emission is the average Θ for the background T=2 and 3 states that can mix into the T=4 state. For this estimate, a shell-model calculation was again carried out by allowing 4p-4h excitations within the $0d_{3/2}, 1s_{1/2}, 0f_{7/2}, 1p_{3/2}$ model space using the interaction of Ref. [11]. In the energy region near the T=4 state, the average allowed spectroscopic factor was found to be $\langle \Theta \rangle \sim 0.03$, which gives an escape width to the lowest T=5/2 state of the order 5 keV. Because of the large separation energy, however, the decay can also occur to higher-lying T=5/2 states. Although the penetrability factor decreases rapidly with the

separation energy, this can easily be compensated for by the density of final states. The escape width is then summed over the possible final states, giving

$$\Gamma^- = \langle \Theta \rangle^2 \int_0^{E_{\max}} dE \rho(E_{\max} - E) \Gamma^-(E) \cup \langle \Theta \rangle^2 74 \text{ MeV} \cup 70 \text{ keV}, \quad (7)$$

where E is the energy of the emitted proton and a Fermi-gas model was used to estimate the density of final states $\rho(E)$.

In addition to proton emission, the $T=4$ state can also decay via the isospin-allowed emission of two-protons to the $T=3$ level in ^{38}Ar . The escape width for this channel can also be estimated using Eq. (2). Using the separation energy 1.6(4) MeV to this level and assuming a spectroscopic factor of unity (worst-case scenario) leads to an escape width of approximately 1 eV. Consequently, the $J=0^+$, $T=4$ state in ^{40}Ca , with an excitation energy of 35.116(412) MeV, is expected to have a decay width of the order 500 keV or less.

CONCLUSIONS

The effects of isospin-symmetry breaking on the emission of protons following β -decay and the possibility of observing high-lying $T=4$ states in even-even, $N=Z$ nuclei was discussed in the present work. For β -delayed proton emission, it was shown that the decay widths for the isospin-forbidden proton decay of the analog state are strongly dependent on the exact locations of the background $T=1$ states that mix into the analog. Consequently, in the absence of experimental information about these states, it is possible to present only a qualitative picture of the decay process. In particular, comparisons were made between experimental values and theoretical estimates for isospin-forbidden spectroscopic factors, which were found to be in agreement within the limits imposed on theory due to uncertainties in the excitation energies of the mixed $T=1$ states. In addition, due to the phase space available to isospin-forbidden proton emission, this decay mechanism is found to be generally favored over the emission of photons.

The excitation energy of the $T=4$ analog state in ^{40}Ca was estimated making use of the IMME, and was found to be bound relative to isospin-allowed proton and neutron emission. Estimates were presented for the spreading and escape widths for this state, leading to a total width of approximately 500 keV.

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